AP Calculus AB – Unit 3 Outline – Basic Derivative Rules

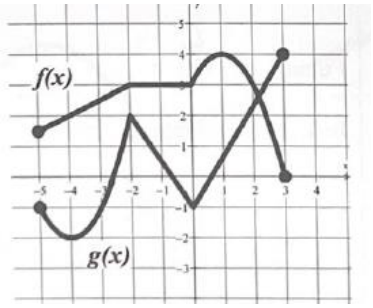
Monday 9/16	**Today's Topic**: **Average Rates of Change**

In-class examples: **Ex. 1** Find the slope of the secant line to the graph of $f(x)=x^2-2x$ from $x=-3$ to $x=1$.

Ex. 2 Find the average rate of change of $f(x)=x^3-x$ over the interval $[1,3]$.

Ex. 3 Find the average rate of change of $f(x)=365e^{-3x}$ over the interval $[0,2]$

Ex. 4 The graphs of *f* and *g* are given below. For each, find the average rate of change on the given interval.



a) $h(x)=f(x)+g(x)$ on $[-4,3]$

b) $k(x)=f(g(x))$ on $[-4,0]$

c) $w(x)=g(f(x))$ on $[-2,3]$

AP Multiple Choice

x	0	2	4	6	8	10
$f(x)$	5	7	8	0	–15	–20

Let f be a differentiable function with selected values given in the table above. What is the average rate of change of f over the closed interval $0 \le x \le 10$?

(A) -6 (B) $-\frac{5}{2}$ (C) -2 (D) $-\frac{2}{5}$ (E) $\frac{2}{5}$

What is the average rate of change of $y=\cos(2x)$ on the interval $\left[0,\frac{\pi}{2}\right]$?

(A) $-\frac{4}{\pi}$ (B) -1 (C) 0 (D) $\frac{\sqrt{2}}{2}$ (E) $\frac{4}{\pi}$

Homework: Worksheet 20

Tuesday 9/17	**Today's Topic**: **The Derivative and Tangent Line Problem;** Derivative as the limit of the difference quotient; Definition of the Derivative = slope of the tangent line to a graph; Finding the slope of a graph or its tangent line at a point; Notations for the derivative.

In-class examples:

Ex. 1 Find the equation of the line that is tangent to the graph of $f(x)=x^2$ at the point $(3,9)$.

Ex. 2 Find $\frac{dy}{dx}$ when $y=2x+3$.

Ex. 3 Find the derivative of $f(x)=x^2-6x+11$

Ex. 4 Find the derivative of $y=\sqrt{x}$.

AP Multiple Choice

None

Homework: Worksheet 21

Wednesday 9/18	**Today's Topic**: **The Power Rule –** In the next few weeks we will be learning how to find the derivative of a number of different types of functions. Today, we will look at constant functions and functions that involve $a \bullet x^n$. **Remember:** Finding the derivative means that we are finding the slope of a function.

In-class examples: Differentiate each function. Label each function appropriately (i.e. y' = ___ or *f'(x)* = ____)

Ex. 1 a) $y=3x^2+2x-1$ b) $s(t)=-4.9t^2+120t+80$

c) $f(x)=4\sqrt{x}-\frac{1}{x}$ d) $h(t)=-5t^{-3}+\frac{3}{\sqrt[3]{t^4}}-5t+4$

Ex. 2 Find the slope of $f(x)=x^3+3x-1$ at $x=2$.

AP Multiple Choice

If $f(x)=x^3-x^2+x-1$, then $f'(2)=$

(A) 10 (B) 9 (C) 7 (D) 5 (E) 3

If $f(x)=\sqrt{x}+\frac{3}{\sqrt{x}}$, then $f'(4)=$

(A) $\frac{1}{16}$ (B) $\frac{5}{16}$ (C) 1 (D) $\frac{7}{2}$ (E) $\frac{49}{4}$

Homework: Worksheet 22

Thursday 9/19	**Today's Topic**: We will be using the **Power Rule** in order to find equations of tangent lines. We will also be determining the points at which a graph has a horizontal tangent line.

In-class examples: **Ex. 1** For the curve $f(x)=x^2+2x+1$:

a) Graph the curve.

b) Find the equation of the tangent line to the curve at the point $(1,4)$. Graph the tangent line.

c) Find the equation of the normal line to the curve at the point $(1,4)$. Graph the normal line.

Ex. 2 Find the equations of the tangent and normal lines to the graph of $f(x)=x^3-3x+2$ when $x=2$.

Ex. 3 For what value(s) of x does $y=x^4-4x^2+1$ have a horizontal tangent line?

AP Multiple Choice

Let f be the function given by $f(x) = x^3 - 6x^2 + 8x - 2$. What is the instantaneous rate of change of f at $x = 3$?

(A) -5 (B) $-\frac{15}{4}$ (C) -1 (D) 6 (E) 17

If $f(x) = 4x^{-2} + \frac{1}{4}x^2 + 4$, then $f'(2) =$

(A) -62 (B) -58 (C) -3 (D) 0 (E) 1

Homework: Worksheet 23

Friday 9/20	**Today's Topic**: **The Package Rule -** We will be using this rule to find the derivate of composite functions in the form of $f(x)=a\cdot[\ \textit{stuff}\ ]^n$, where the $[\ \textit{stuff}\ ]$ is some function other than x.

In-class examples: Find the derivative of each of the following:

Ex. 1 $f(x)=(3x^2-4x+1)^3$ **Ex. 2** $y=\left(\frac{1}{2}x^2-x^{\frac{1}{2}}\right)^3$ **Ex. 3** $f(x)=\sqrt[3]{(x^2-1)^5}$ **Ex. 4** $f(x)=\sin^3 x$

Ex. 5 Find the equation of the normal line to $f(x)=\sqrt{x-1}$ when $x=5$.

AP Multiple Choice

If $f(x) = \sqrt{x^2 - 4}$ and $g(x) = 3x - 2$, then the derivative of $f(g(x))$ at $x = 3$ is

(A) $\frac{7}{\sqrt{5}}$ (B) $\frac{14}{\sqrt{5}}$ (C) $\frac{18}{\sqrt{5}}$ (D) $\frac{15}{\sqrt{21}}$ (E) $\frac{30}{\sqrt{21}}$

Homework: Worksheet 24

Monday 9/23	**Today's Topic**: **Product and Quotient Rules** "Boxes and Triangles" - To find the derivative of a function using the **Product and Quotient Rules**.

In-class examples:

Ex. 1 Find $\frac{dy}{dx}$ if $y=(2x+1)(x^2-2)$

Ex. 2 Find $f'(x)$ if $f(x)=\frac{x^2-1}{x^2+3x+2}$

Ex. 3 Find $f'(x)$ for $f(x)=x^2(x-2)^4$.

AP Multiple Choice

$$\frac{d}{dx}\left(\frac{x+1}{x^2+1}\right)=$$

(A) $\frac{x^2+2x-1}{(x^2+1)^2}$ (B) $\frac{-x^2-2x+1}{x^2+1}$ (C) $\frac{-x^2-2x+1}{(x^2+1)^2}$ (D) $\frac{3x^2+2x+1}{(x^2+1)^2}$ (E) $\frac{1}{2x}$

Homework: Worksheet 25

Tuesday 9/24	**Today's Topic**: **Derivatives Using Tables**

In-class examples:

Ex. 1 Let $f(x)$ and $g(x)$ be differentiable functions with the values for $f(x)$, $g(x)$, $f'(x)$, and $g'(x)$ as shown in the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	-2	3	1
3	10	-4	4	7
4	-1	-3	6	8

Determine each of the following values:

a) $h(x)=f(x)+g(x)$. Find $h'(1)$.

b) $p(x)=f(x)\cdot g(x)$. Find $p'(1)$.

c) $q(x)=\frac{f(x)}{g(x)}$. Find $q'(3)$.

d) $r(x)=[f(x)]^4$. Find $r'(1)$.

e) $c(x)=f(g(x))$. Find $c'(3)$.

AP Multiple Choice

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	3	4	2	π

The table above gives values of the differentiable functions f and g and their derivatives at $x = 0$.

If $h(x)=\frac{f(x)}{g(x)}$, what is the value of $h'(0)$?

(A) $\frac{8-3\pi}{4}$ (B) $\frac{3\pi-8}{4}$ (C) $\frac{4}{\pi}$ (D) $\frac{2-3\pi}{2}$ (E) $\frac{8+3\pi}{4}$

Homework: Worksheet 27

Wednesday 9/25	**Today's Topic**: **Quiz** - Avg ROC, Derivative (Limit Definition), Power Rule, Equation of Tangent Line
In-class Example: None	
Homework: Worksheet 29	

Thursday 9/26 and Friday 9/27	**Today's Topic**: Derivatives Using a Calculator

In-class Example: For each problem, estimate the derivative using a calculator.

1. $f(x) = x^3 - 3x + 7$, find $f'\left(\frac{2}{3}\right)$.
2. $f(x) = \sin\sqrt{x}$, find $f'(2)$.
3. $f(x) = \ln\left(\frac{1}{5-x}\right)$, find $f'(1.3)$.

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4. Write the equation of the line tangent to $y = \sqrt{\frac{x}{x^3+1}}$ at $x = 1$

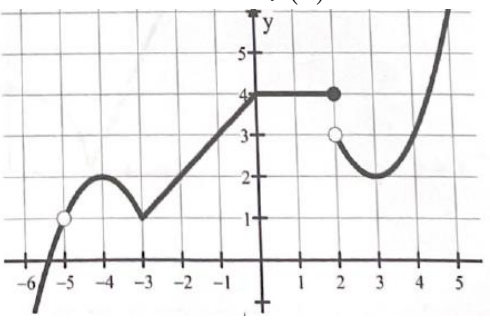
Homework: Worksheet 26

Monday 10/30	**Today's Topic**: **Differentiability:** The term "differentiable" means that a function's derivative can be found. Some functions will not be differentiable at certain values of *x*.

In-class examples:

Ex. 1 The graph of $f(x)$ is given below.

State the values at which $f(x)$ is **(a)** not continuous and **(b)** continuous but not differentiable.



Ex. 2 State the values for which $f(x) = 2x^2 - 4x$ is differentiable.

Ex. 3 State the values for which $f(x) = |x|$ is differentiable.

Ex. 4 For the function, $f(x) = x^{2/3}$, determine if $f(x)$ is differentiable at $x = 0$.

Ex. 5 Determine if $f(x) = \begin{cases} x^2, & x < 1 \\ -2x + 3, & x \geq 1 \end{cases}$ is differentiable at $x = 1$.

AP Multiple Choice

$$f(x) = \begin{cases} 3x + 5 & \text{when } x < -1 \\ -x^2 + 3 & \text{when } x \geq -1 \end{cases}$$

If f is the function defined above, then $f'(-1)$ is

(A) -3 (B) -2 (C) 2 (D) 3 (E) nonexistent

Homework: Worksheet 28

Tuesday 10/1	**Today's Topic**: Review for our test.

In-class examples: None

AP Multiple Choice

Let f be a differentiable function such that $f(0) = -5$ and $f'(x) \leq 3$ for all x. Of the following, which is not a possible value for $f(2)$?

(A) -10 (B) -5 (C) 0 (D) 1 (E) 2

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x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	-2	-3	4

The table above gives values of the differentiable functions f and g and their derivatives at $x = 1$. If $h(x) = (2f(x) + 3)(1 + g(x))$, then $h'(1) =$

(A) -28 (B) -16 (C) 40 (D) 44 (E) 47

If $y = 5x\sqrt{x^2 + 1}$, then $\frac{dy}{dx}$ at $x = 3$ is

(A) $\frac{5}{2\sqrt{10}}$ (B) $\frac{15}{\sqrt{10}}$ (C) $\frac{15}{2\sqrt{10}} + 5\sqrt{10}$ (D) $\frac{45}{\sqrt{10}} + 5\sqrt{10}$ (E) $\frac{45}{\sqrt{10}} + 15\sqrt{10}$

Homework: Worksheet 30

Wednesday 10/2	**Today's Topic**: Unit 2 Exam – Basic Differentiation Rules

In-class examples: None

Homework: None